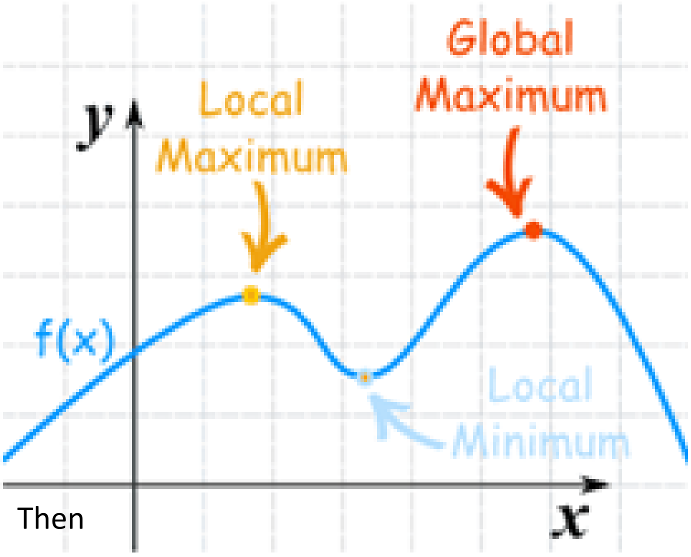
4-1 Extreme Value of a Function

<u>Learning Targets</u>

I can identify relative and absolute extrema from a graph.

I can apply the Extreme Value Theorem to identify absolute extrema on a closed interval.

I can identify all critical points for a function.



Absolute Extrema: (Global max or min)

Let f(x) be a function with domain D. Then

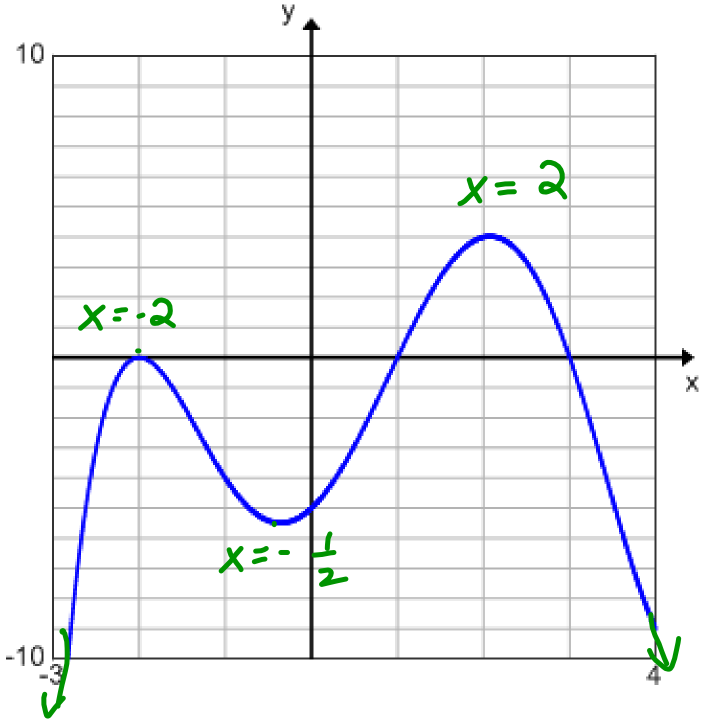
- f(c) is the absolute maximum value on D if and only if $f(x) \le f(c)$ for all x in D.
- f(c) is the absolute minimum value on D if and only if $f(x) \ge f(c)$ for all x in D.

Local Extrema: (relative max or min)

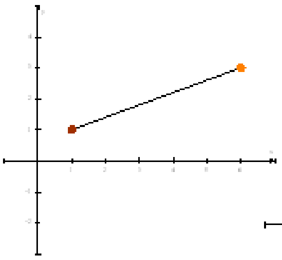
Let f(x) be a function defined on an open interval containing c. Then

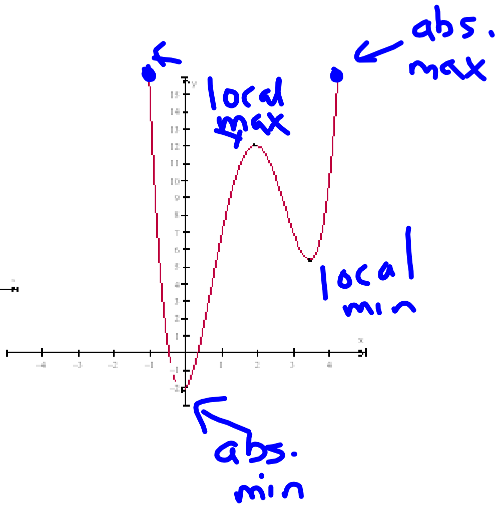
- f(c) is a local maximum value if and only if $f(x) \le f(c)$ for all x in an open interval containing c.
- f(c) is a local minimum value if and only if $f(x) \ge f(c)$ for all x in an open interval containing c.

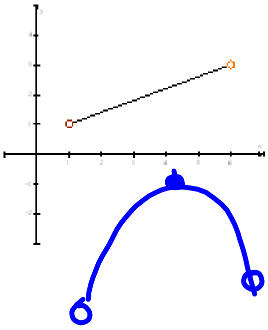
Derivatives help us locate max's and min's because the slopes of the tangent line at a max or min value = 0.



On a closed interval, the max's or min's can occur at interior points OR at an endpoint.





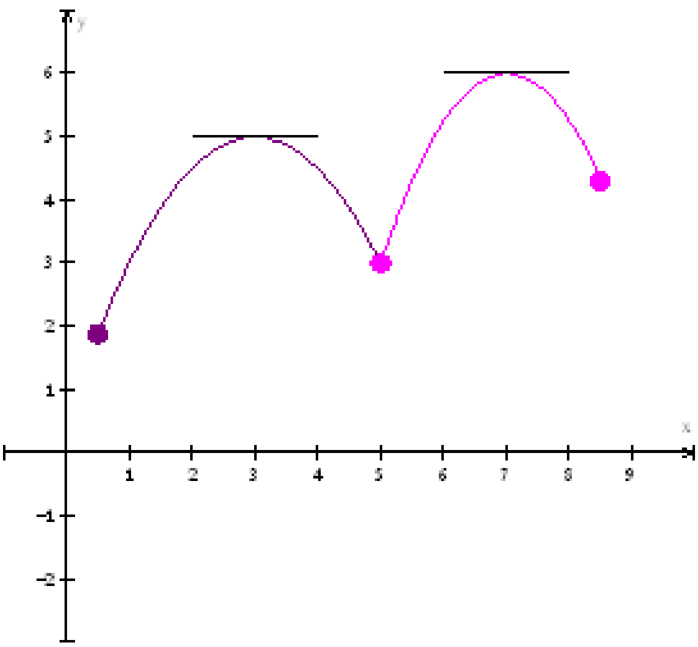


A max or min can exist at an endpoint on a function defined on a closed interval even though f'(x) does not exist at that point.

The same is not true on an open interval. An abs max or a min may or may not exist. A max or min cannot exist at an endpoint on an open interval because there is not an endpoint.

Critical Point

A point is a critical point if f'(x)=0 (max or min) or if f'(x) doesn't exist (sharp corner, discontinuity, vertical tangent line).



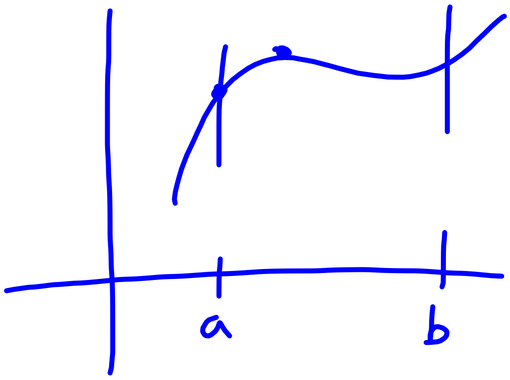
places where the derivative = 0

endpoints

places where the derivative does not exist

Extreme Value Theorem

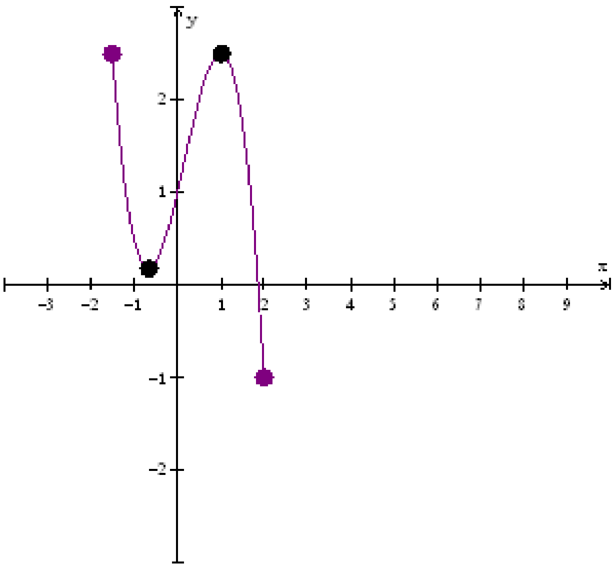
If f(x) is continuous on a closed interval [a, b], then f <u>must</u> have both an absolute max and min on that interval.



Oct 15-11:20 AM

Decide if the function meets the conditions of the extreme value theorem. If it does, identify the absolute max and absolute min.

1.



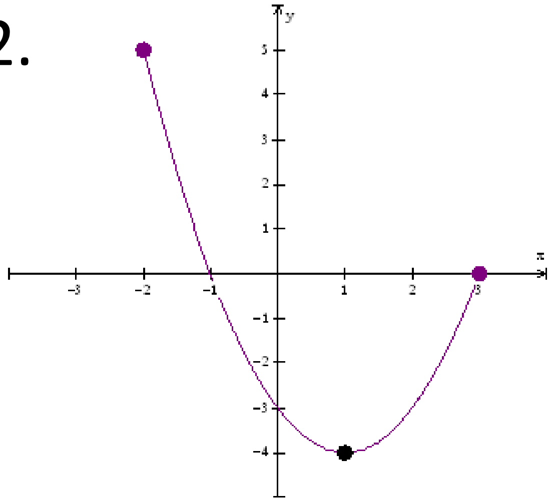
meets EVT

min $x = 2$

max $x = -2$
$x = 1$

Oct 15-11:20 AM

2.



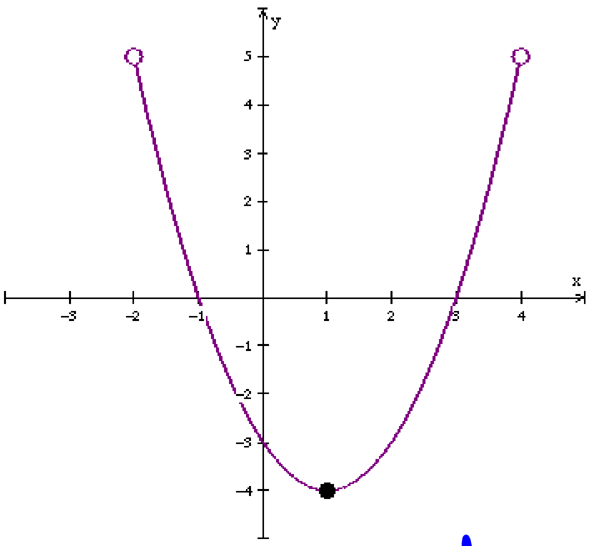
meets EVT (continuous/closed interval)

$x = -2$ max

$x = 1$ min

Oct 15-11:20 AM

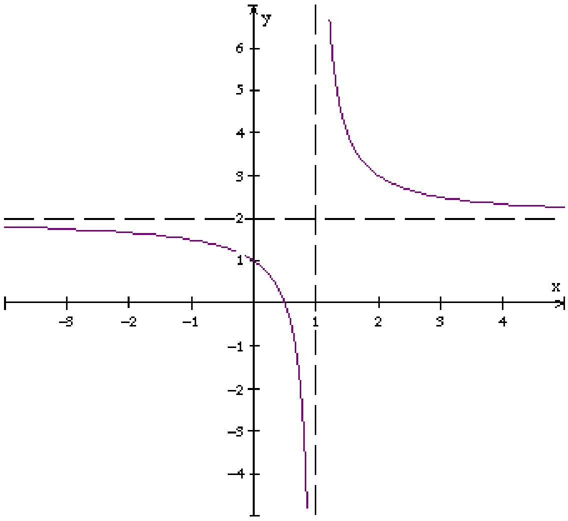
3.



does not meet EVT
(not closed)

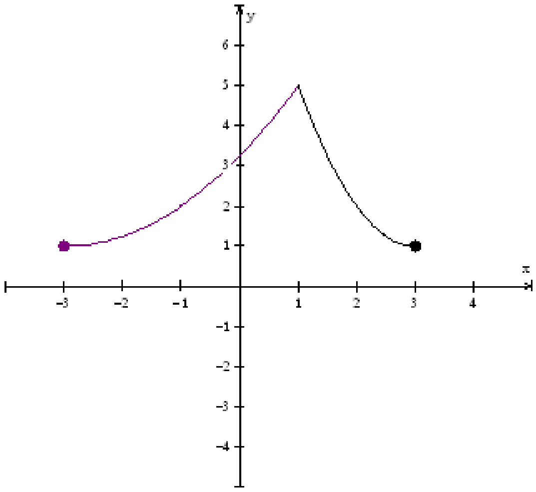
still has abs. min at x = -1 > doesn't have to
but no max

4.



does not meet EVT
(not continuous)

5.



meets EVT (continuous/closed)

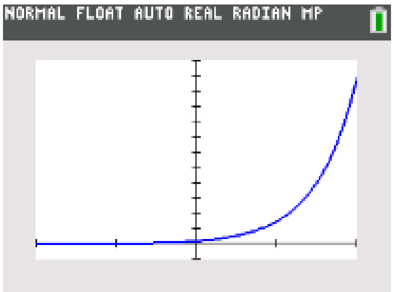
abs max : $x = 1$

abs min : $x = -3, x = 3$

Oct 15-11:20 AM

Decide if the extreme value theorem applies to the given function. Find the absolute extrema (if they exist).

1. $f(x) = e^{2x}, -2 \le x \le 2$



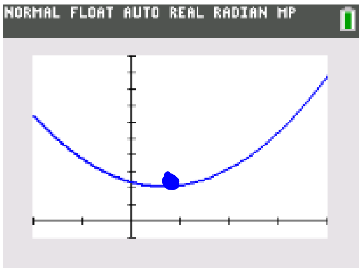
meets EVT

abs min: $(-2, e^{-4})$ .0183

abs max: $(2, e^{4})$ → 54.598

Oct 15-11:20 AM

2. $y = 3x^2 - 4x + 12$ on $[-2, 4]$



Meets EVT
(continuous on a closed interval)

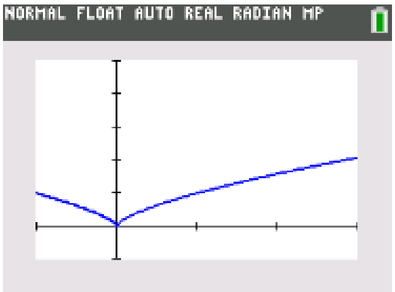
abs min: $\left(\frac{2}{3}, 10.667\right)$ (.667)

abs max: $(4, 44)$

$y' = 6x - 4 = 0$

$x = 2/3$

3. $g(x) = x^{\frac{2}{3}}$ on $[-1, 3]$



meets EVT

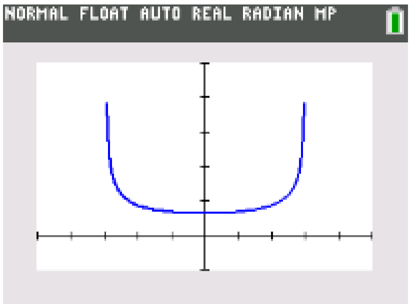
2.080

abs max: $(3, 3^{2/3})$

abs min: $(0, 0)$

Oct 15-11:20 AM

4. $h(x)=\dfrac{2}{\sqrt{9-x^2}}$



does not meet EVT
(open interval)
min: $(0, \frac{2}{3})$

Oct 15-11:20 AM

5. Find the extreme values of

$$f(x)=\begin{cases} x^2-5 & x\ge -2 \\ 1+x & x<-2 \end{cases}$$

$y=x^2-5$

x	y
-2	-1
-1	-4
0	-5

$y=1+x$

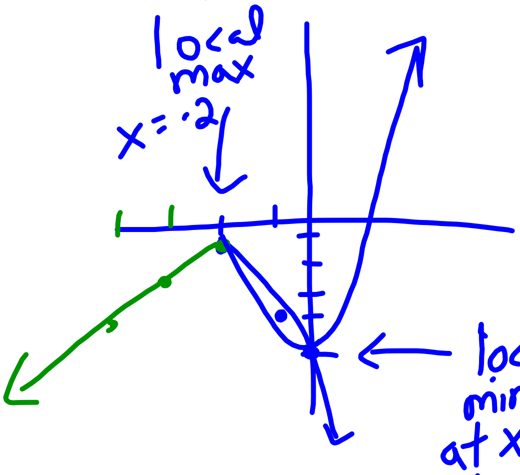
x	y
-2	-1
-3	-2
-4	-3

does not meet EVT

(-2, -1)

local max

x = -2



local min at x=0

(0, -5)

Oct 15-11:20 AM

Homework

p. 193 #1-11, 13, 14, 17, 18, 25, 31, 32, 45-50